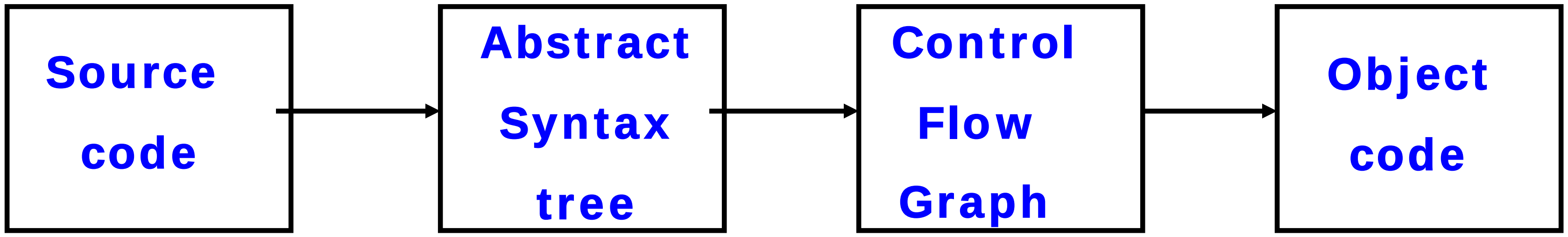
Data Flow Analysis 1

Compiler Design

Compiler Structure



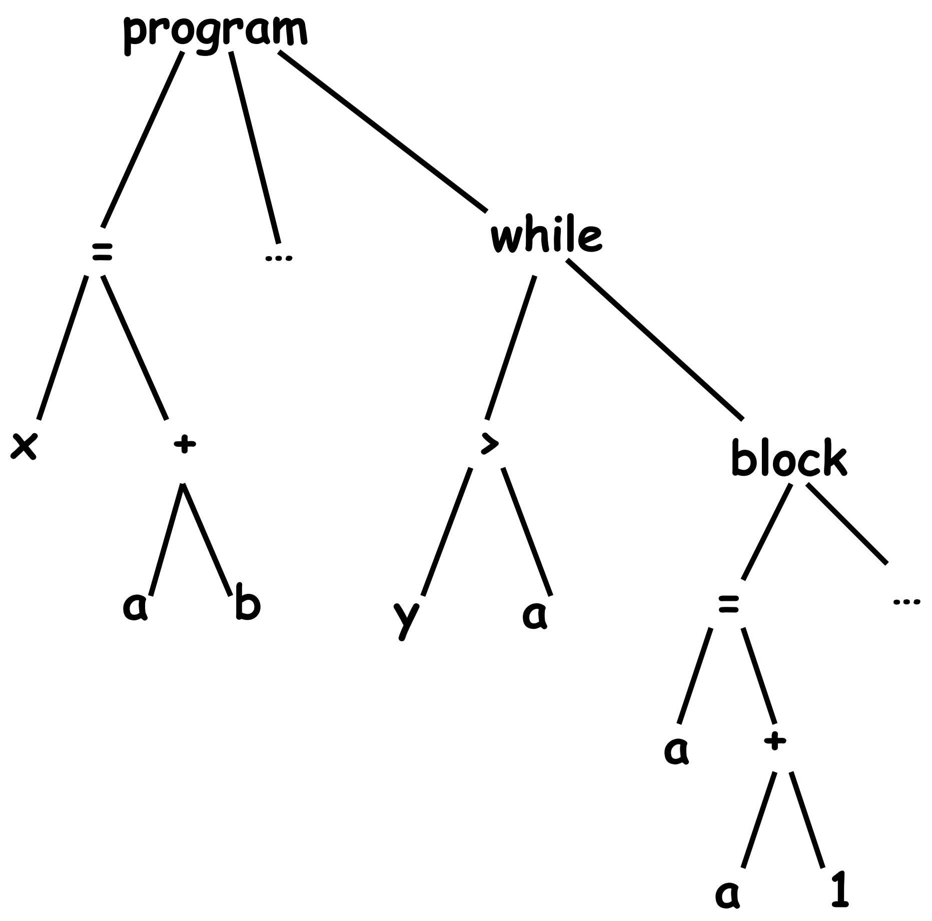
- Source code parsed to produce abstract syntax tree.
- Abstract syntax tree transformed to *control flow graph*.
- *Data flow analysis* operates on the control flow graph (and other intermediate representations).

Abstract Syntax Tree (AST)

- Programs are written in text
    - as sequences of characters
    - may be awkward to work with.
- First step: Convert to structured representation.
    - Use lexer (like lex) to recognize tokens
    - Use parser (like yacc) to group tokens structurally
        - often produce to produce AST

Abstract Syntax Tree Example

```
x := a + b;

y := a * b

While (y > a){

   a := a +1;

   x := a + b

}
```



ASTs

- ASTs are abstract
  - don’t contain all information in the program
    - e.g., spacing, comments, brackets, parenthesis.
  - Any ambiguity has been resolved
    - e.g., a + b + c produces the same AST as (a +b) + c.

Disadvantages of ASTs

- ASTs have many similar forms
  - e.g., for while, repeat , until, etc
  - e.g., if, ?, switch
- Expressions in AST may be complex, nested

```
(42 * y) + ( z > 5 ? 12 * z : z +20)
```

- Want simpler representation for analysis
  - ... at least for dataflow analysis.

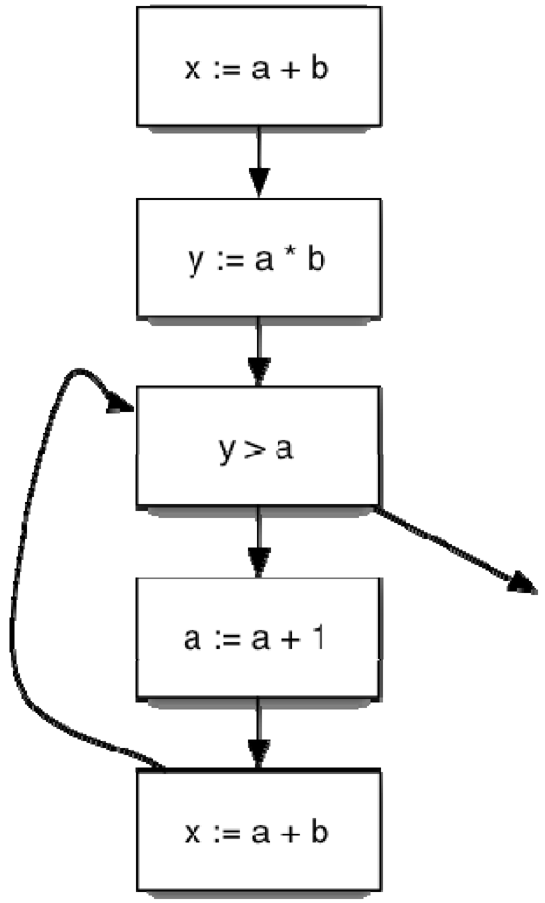
Control-Flow Graph (CFG)

- A directed graph where
  - Each node represents a statement
  - Edges represent control flow

- Statements may be
  - Assignments x = y op z or x = op z
  - Copy statements x = y
  - Branches goto L or if relop y goto L
  - etc

Control-flow Graph Example

```
x := a + b;

y := a * b

While (y > a){

  a := a +1;

  x := a + b

}
```

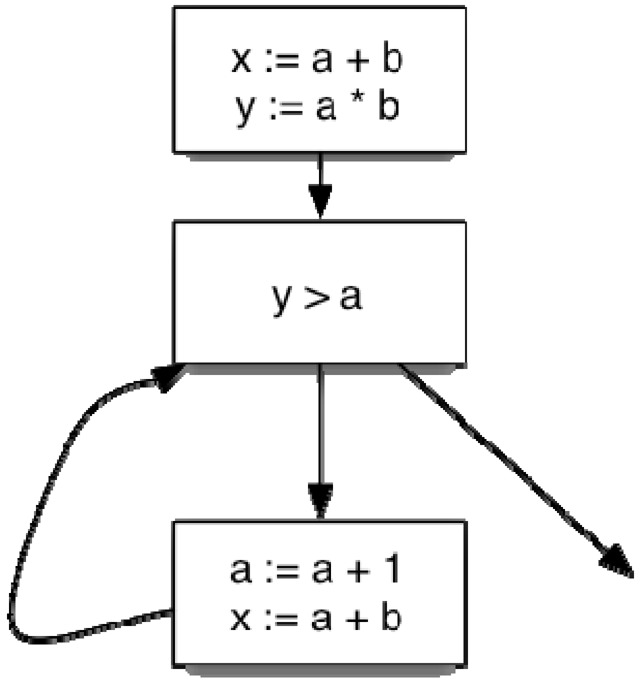


Variations on CFGs

- Usually don’t include declarations (e.g. int x;).
- May want a unique entry and exit point.
- May group statements into *basic blocks.*
  - A *basic block* is a sequence of instructions with no branches into or out of the block.

Control-Flow Graph with Basic Blocks

```
X := a + b;

Y := a * b

While (y > a){

   a := a +1;

   x := a + b

}
```



- Can lead to more efficient implementations
- But more complicated to explain so...
  - We will use single-statement blocks in lecture

CFG vs. AST

- CFGs are much simpler than ASTs
  - Fewer forms, less redundancy, only simple expressions
- But, ASTs are a more faithful representation
  - CFGs introduce temporaries
  - Lose block structure of program
- So for AST,
  - Easier to report error + other messages
  - Easier to explain to programmer
  - Easier to unparse to produce readable code

Data Flow Analysis

- A framework for proving facts about program
- Reasons about lots of little facts
- Little or no interaction between facts
  - Works best on properties about how program computes
- Based on all paths through program
  - including infeasible paths

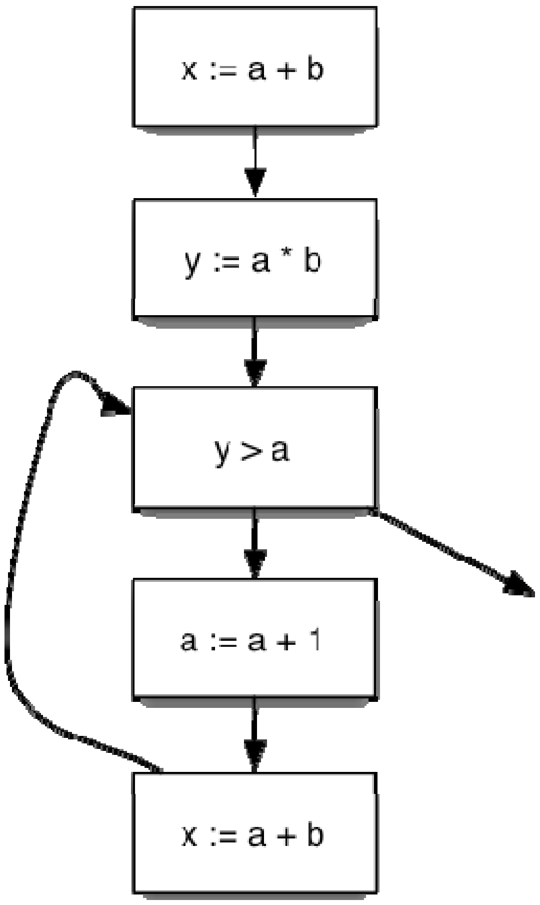
Available Expressions

- An expression e = x op y is *available* at a program point p, if
  - on every path from the entry node of the graph to node p, e is computed at least once, and
  - And there are no definitions of x or y since the most recent occurance of e on the path

- Optimization
  - If an expression is available, it need not be recomputed
  - At least, if it is in a register somewhere

Data Flow Facts

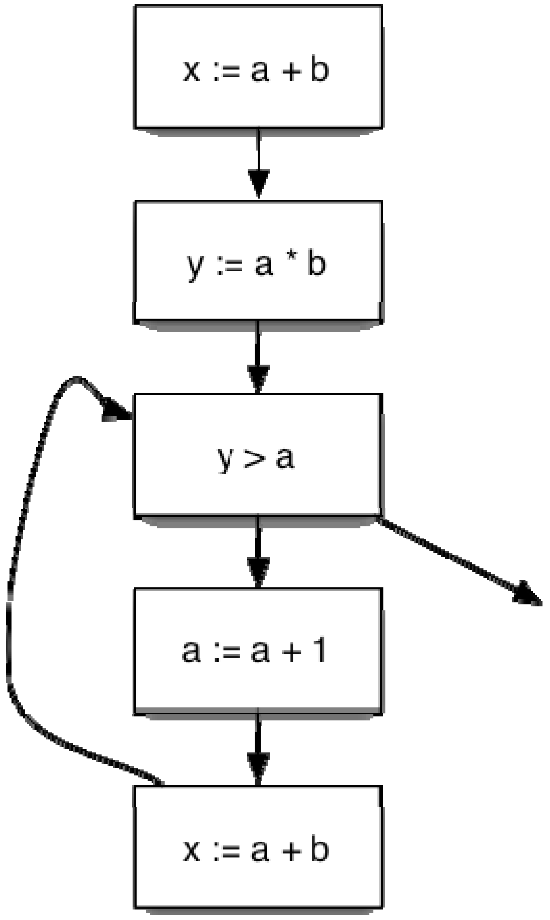
- Is expression e available?
- Facts:
  - a + b is available
  - a * b is available
  - a + 1 is available



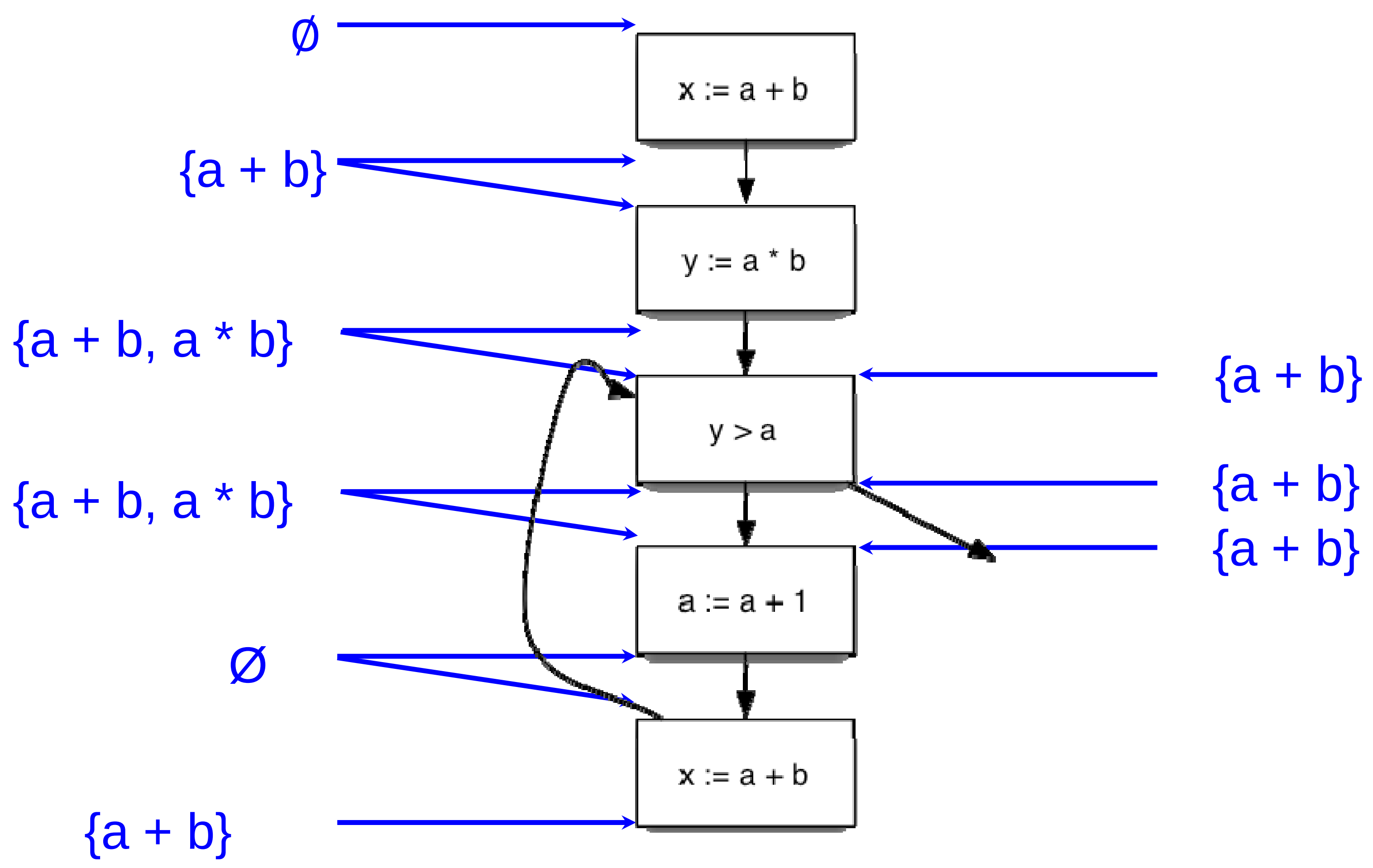
Gen and Kill

What is the effect of each statement on the set of facts?

stmt	gen	kill
x = a + b	a + b	
y = a * b	a * b	
a = a + 1		a + b a * b a + 1



Computing Available Expressions



Terminology

- A *join point* is a program point where two branches meet
- Available expressions is a *forward, must problem*
  - *Forward* = Data Flow from in to out
  - *Must* = At joint point, property must hold on all paths that are joined.

Data Flow Equations

- Let s be a statement
  - succ(s) = {immediate successor statements of s}
  - Pred(s) = {immediate predecessor statements of s}
  - In(s) program point just before executing s
  - Out(s) = program point just after executing s
- In(s) = $\bigcap_{s' \, 5 \, \mathbf{pred(s)}}$ Out(s')
- Out(s) = Gen(s) ^ (In(s) – Kill(s))
  - Note these are also called transfer functions

Liveness Analysis

- A variable v is *live at a program* point p if
  - v will be used on some execution path originating from p before v is overwritten
- Optimization
  - If a variable is not live, no need to keep it in a register
  - If a variable is dead at assignment, can eliminate assignment.

Data Flow Equations

- Available expressions is a forward must analysis
  - Data flow propagate in same direction as CFG edges
  - Expression is available if available on all paths

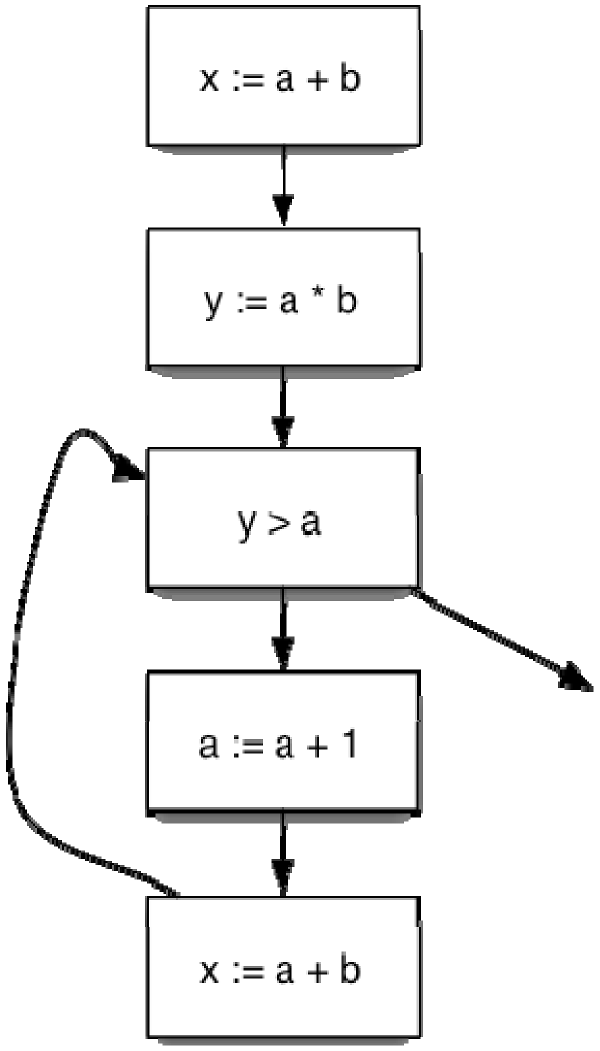
- Liveness is a backward may problem
  - to kow if variable is live, need to look at future uses
  - Variable is live if available on some path

- In(s) = Gen(s) ^ (Out(s) – Kill(s))
- $Out(s) = \bigcup_{\mathbf{s'}\ 5\ \mathbf{succ(s)}} In(s')$

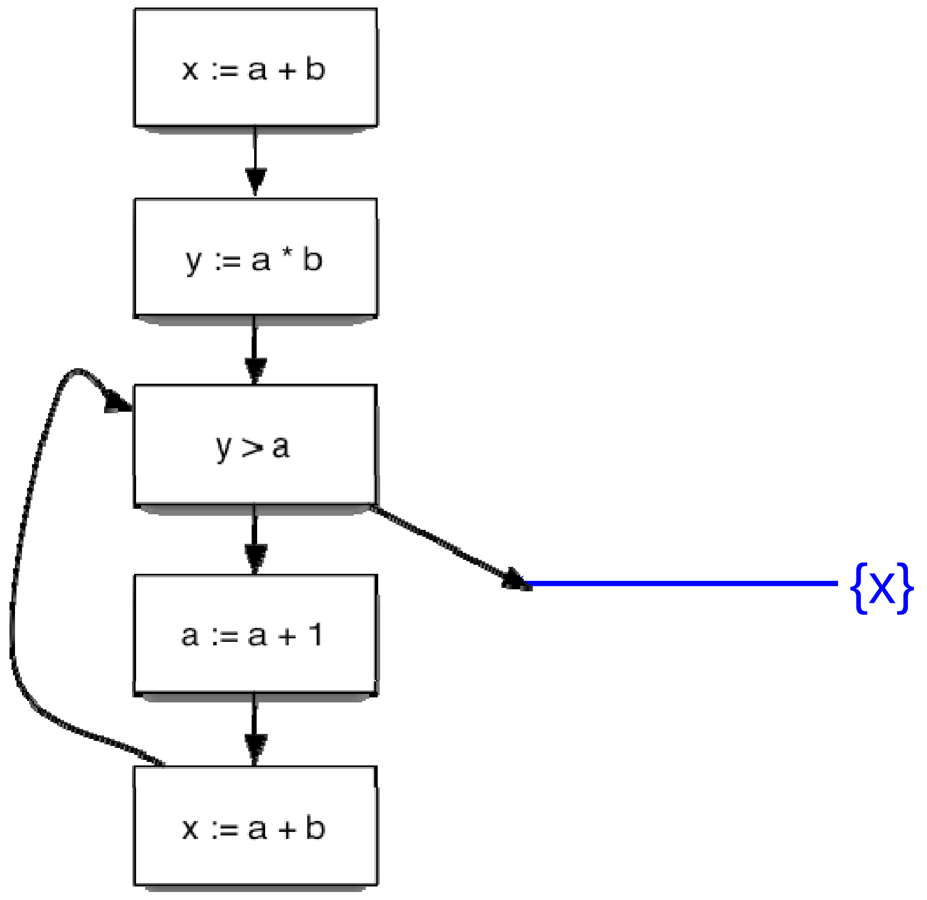
Gen and Kill

What is the effect of each statement on the set of facts?

stmt	gen	kill
x = a + b	a, b	x
y = a * b	a, b	y
y > a	a, y	
a = a + 1	a	a



Computing Live Variables



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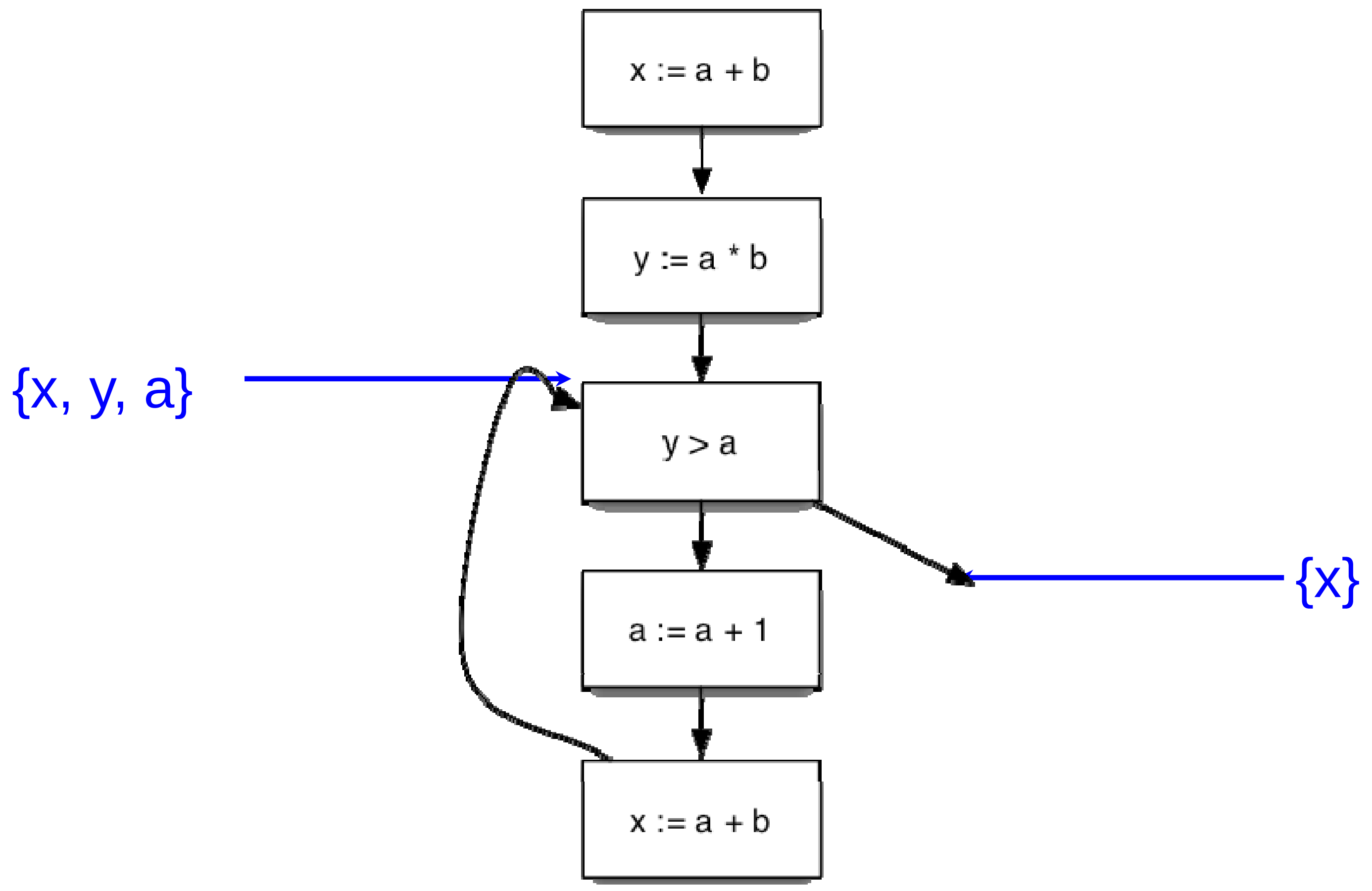

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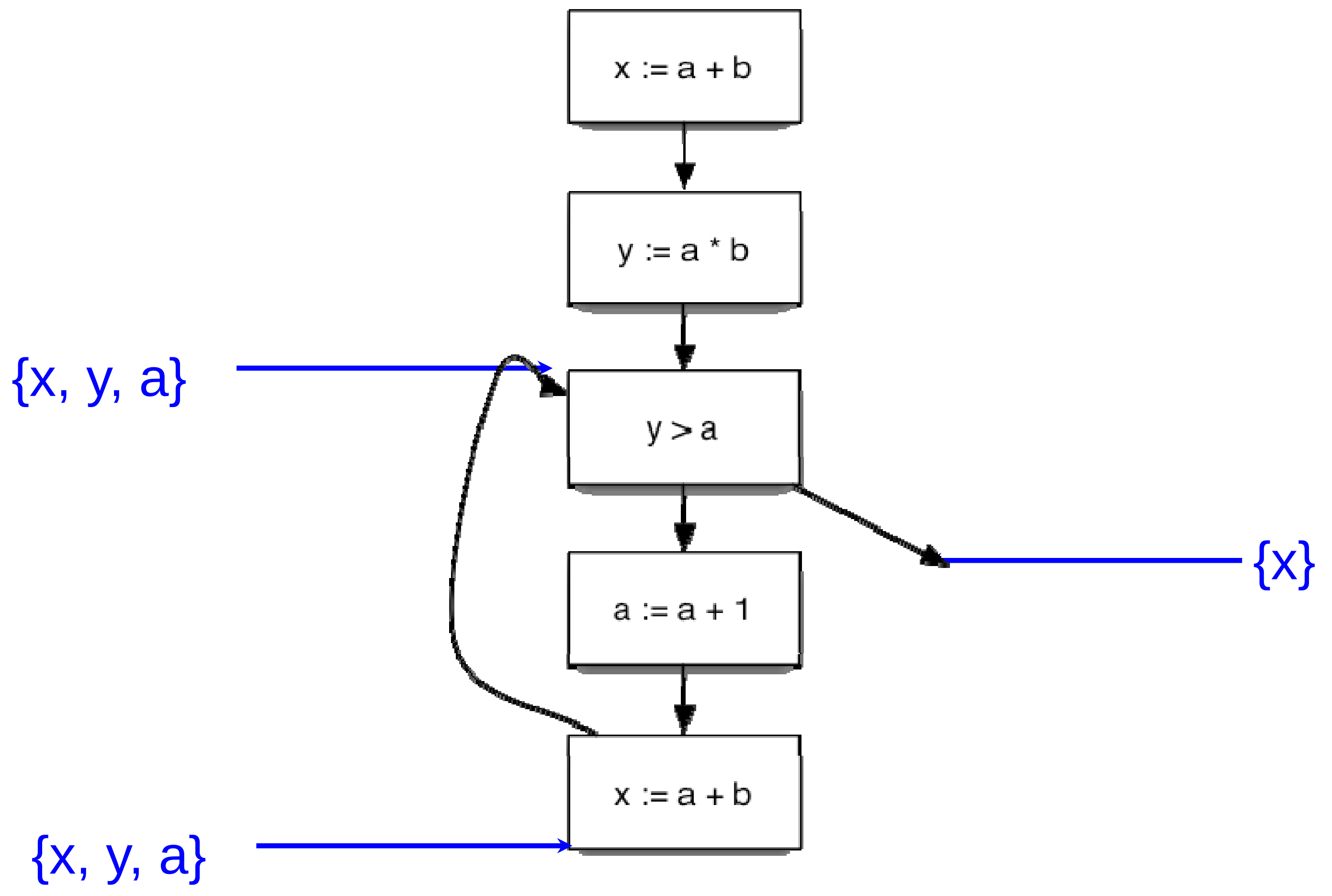

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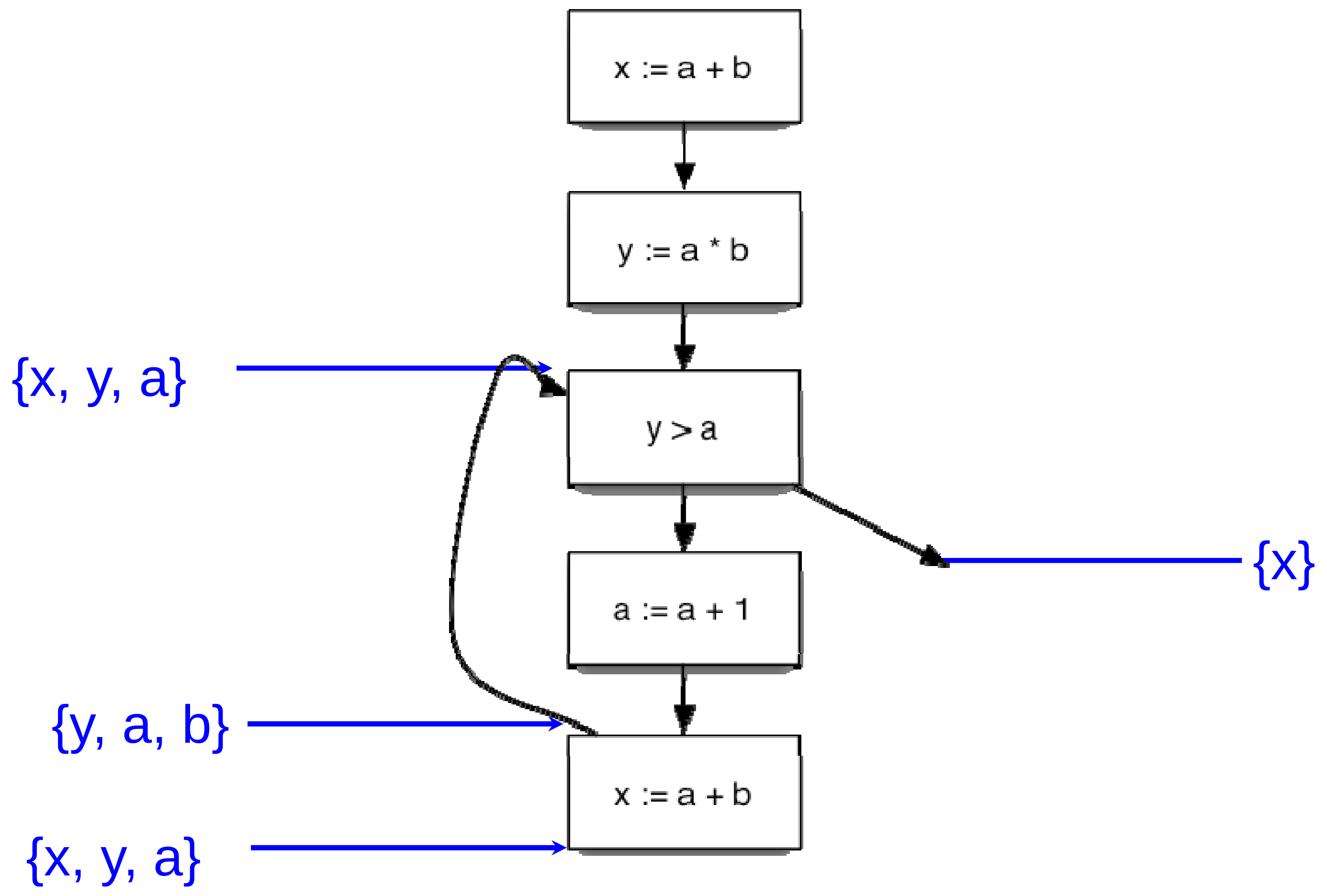

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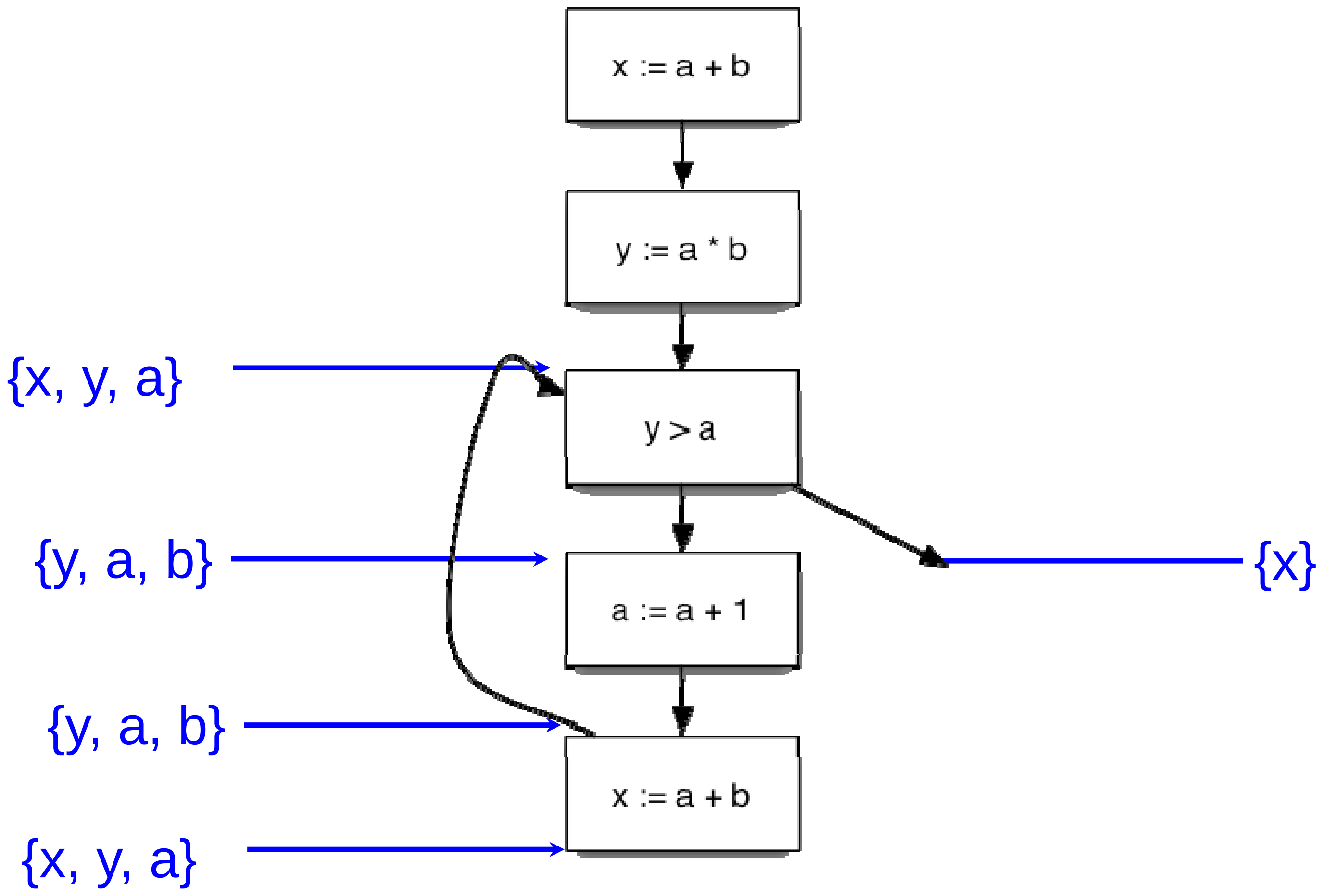

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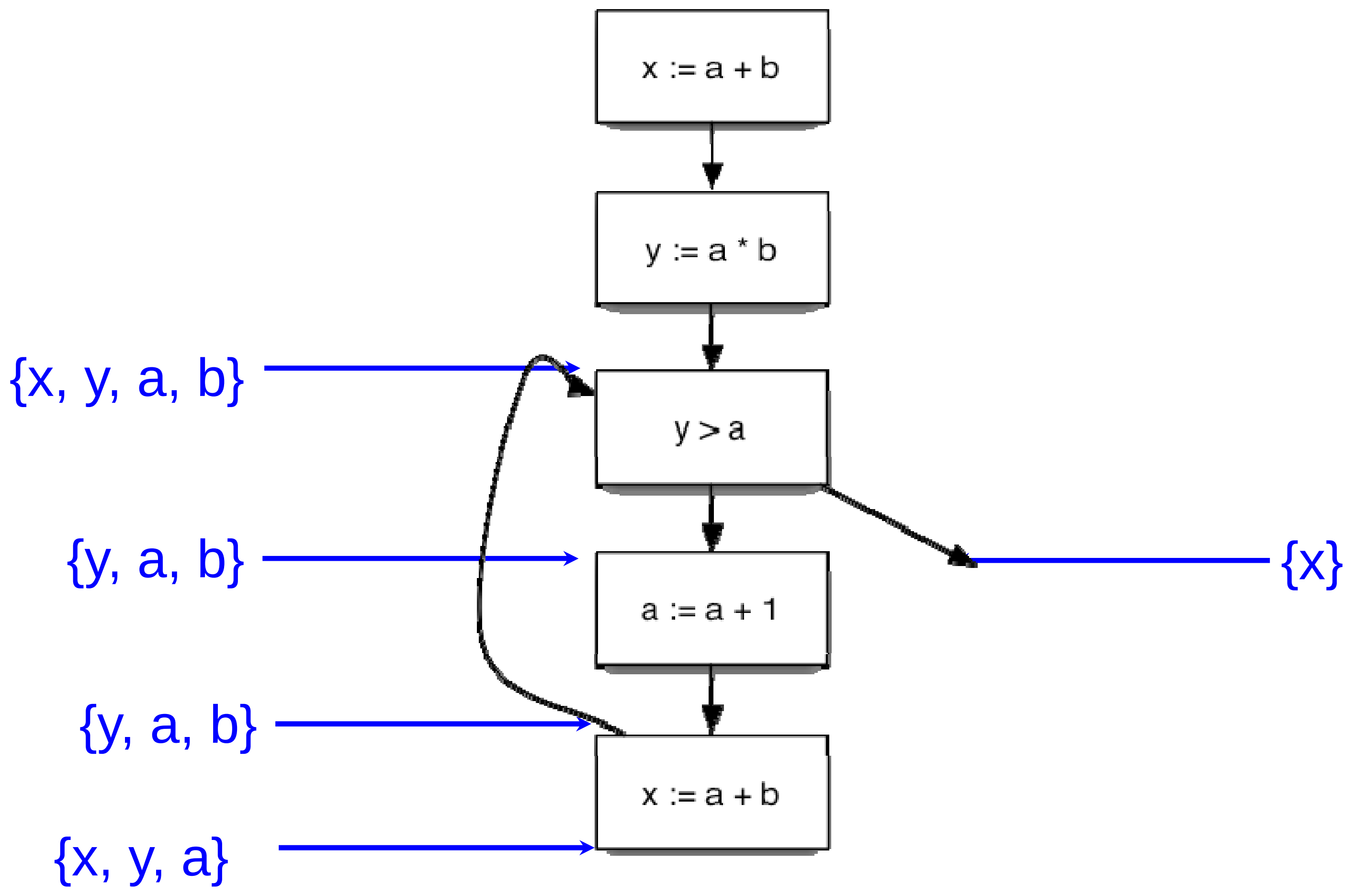

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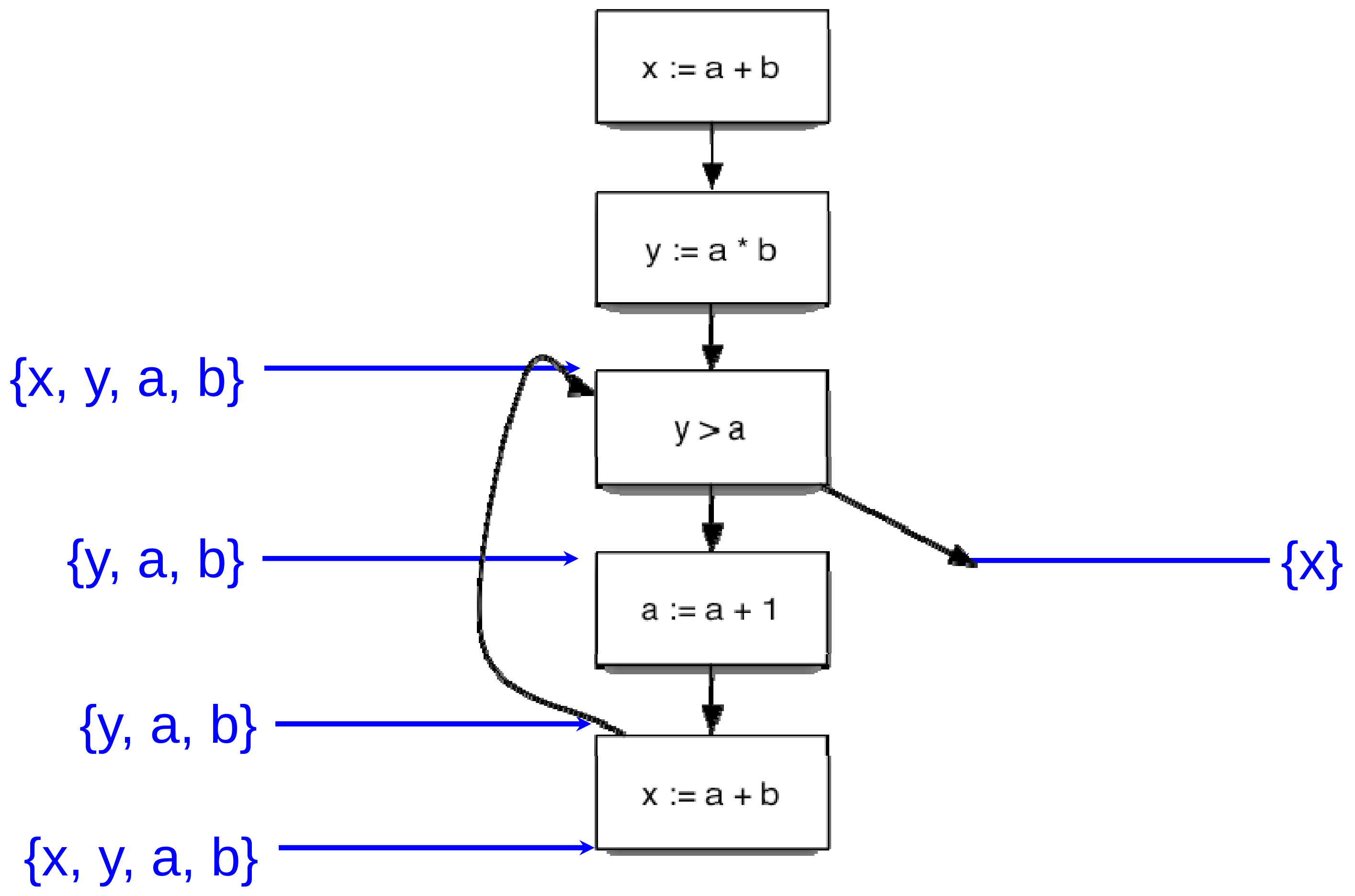

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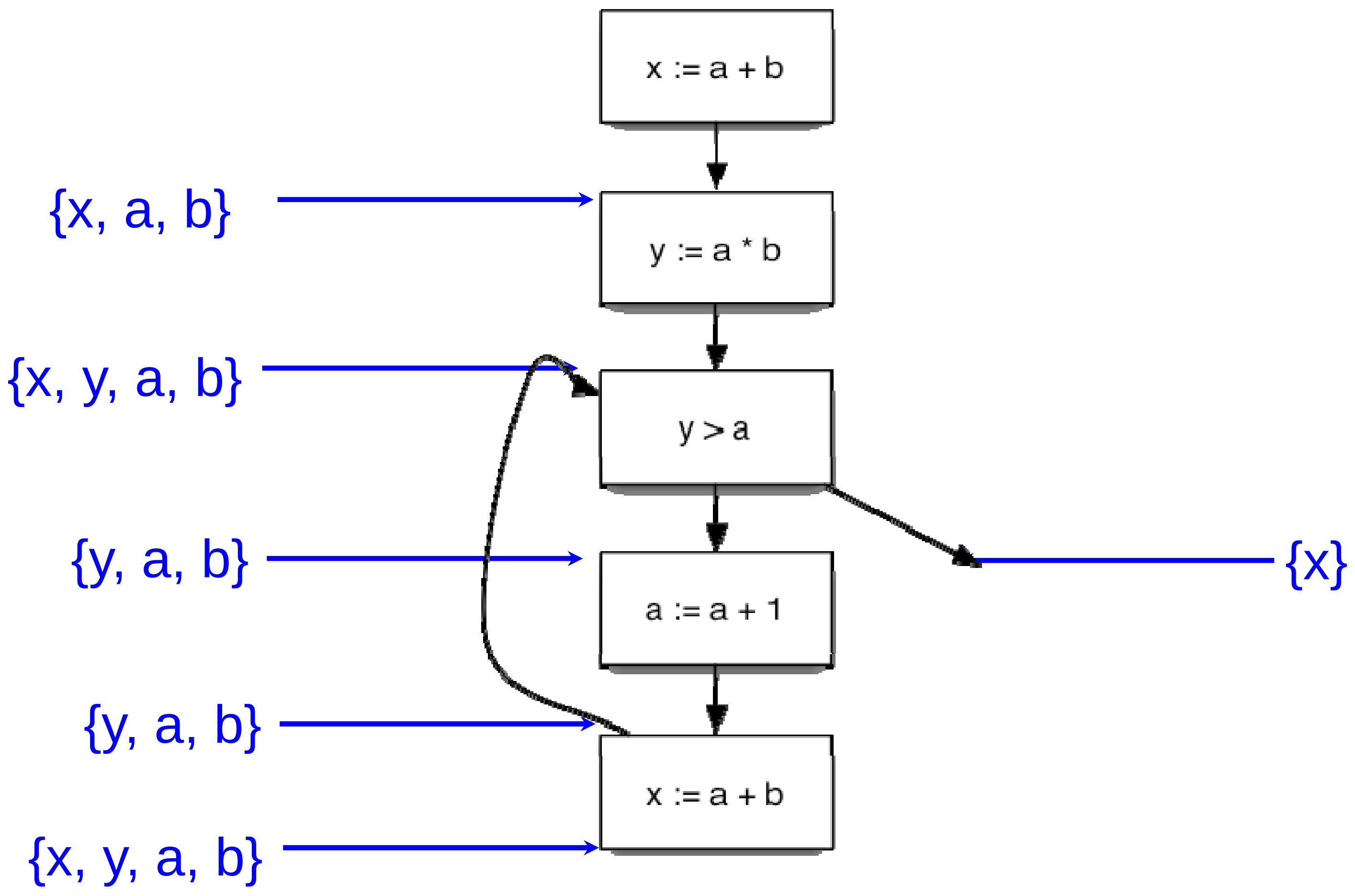

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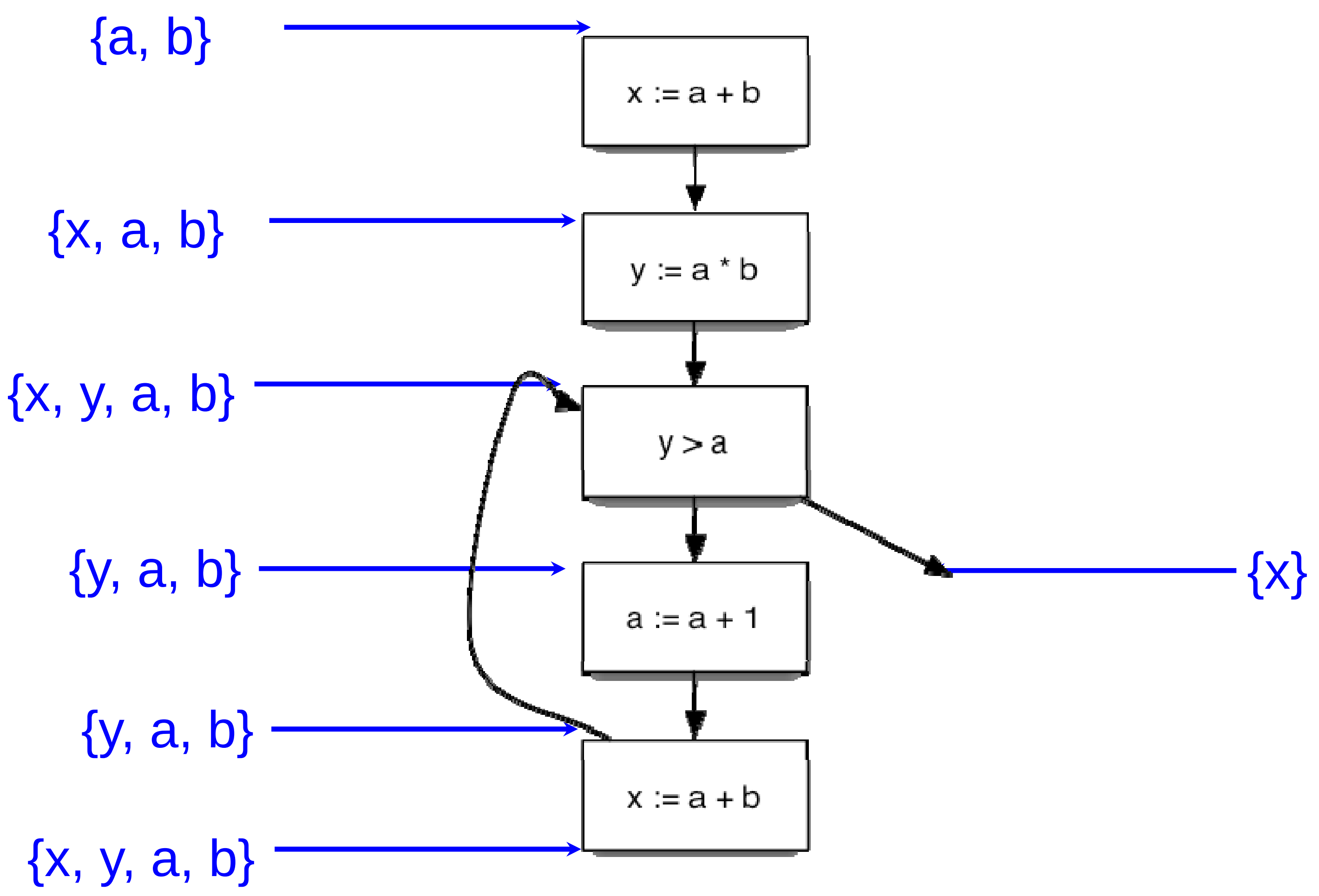


Very Busy Expressions

- An expression e is *very busy at point p* if
  - On every path from p, e is evaluated before the value of e is changed
- Optimization
  - Can hoist very busy expression computation
- What kind of problem?
  - Forward or backward? Backward
  - May or must? Must

Code Hoisting

- Code hoisting finds expressions that are always evaluated following some point in a program, regardless of the execution path and moves them to the latest point beyond which they would always be evaluated.

- It is a transformation that almost always reduces the space occupied but that may affect its execution time positively or not at all.

Reaching Definitions

- A *definition of a variable* v is an assignment to v
- A definition of variable v reaches point p if
  - There is no intervening assignment to v
- Also called *def-use* information
- What kind of problem?
  - Forward or backward? Forward
  - May or must? may

Space of Data Flow Analyses

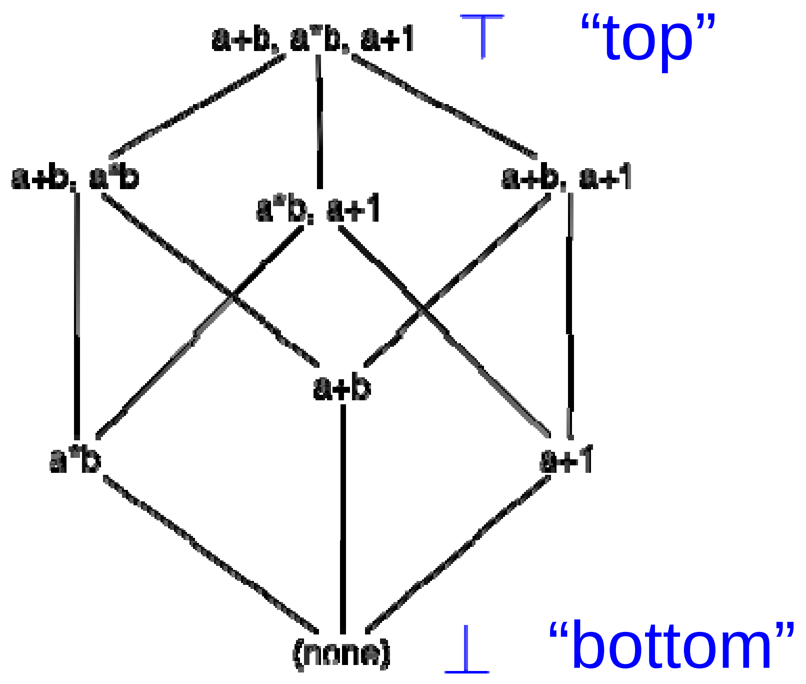
	May	Must
Forward	Reaching definitions	Available expressions
Backward	Live Variables	Very busy expressions

- Most data flow analyses can be classified this way
  - A few don't fit: bidirectional
- Lots of literature on data flow analysis

Data Flow Facts and lattices

Typically, data flow facts form a lattice

Example, Available expressions



Partial Orders

- A *partial order* is a pair (P, ‰) such that
  - ‰ » P § P
  - ‰ is *reflexive*: x ‰ x
  - ‰ is *anti-symmetric*: x ‰ y and y ‰ x implies x = y
  - ‰ is *transitive*: x ‰ y and y ‰ z implies x ‰ z

Lattices

- A partial order is a lattice if x and w are defined so that
  - x is the meet or greatest lower bound operation
    - x x y ‰ x and x x y ‰ y
    - If z ‰ x and z ‰ y then z ‰ x x y
  - w is the join or least upper bound operation
    - x ‰ x w y and y ‰ x w y
    - If x ‰ z and y ‰ z, then x w y ‰ z

Lattices (cont.)

A finite partial order is a lattice if meet and join exist for every pair of elements

A lattice has unique elements bot and top such that

x x B = B   x w B =x

x x A = x   x w A = A

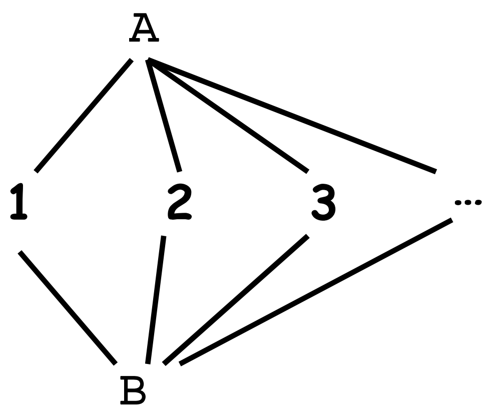
In a lattice

x ‰ y iff x x y = x

x ‰ y iff x w y = y

Useful Lattices

- $(2^S$, ») forms a lattice for any set S.
  - 2^S is the powerset of S (set of all subsets)
- If (S, ‰) is a lattice, so is (S,¿)
  - i.e., lattices can be flipped
- The lattice for constant propagation



Forward Must Data Flow Algorithm

```
Out(s) = Gen(s) for all statements s
W = {all statements} (worklist)
Repeat
        Take s from W
        In(s) = $\bigcap_{\mathbf{s'}\ 5\ \mathbf{pred(s)}}$ Out(s')
        Temp = Gen(s) ^ (In(s) – Kill(s))
        If (temp != Out (s)) {
        Out(s) = temp
        W = W ^ succ(s)
        }
Until W = ∅
```

Monotonicity

- A function f on a partial order is monotonic if

x ‰ y implies f(x) ‰ f(y)

- Easy to check that operations to compute In and Out are monotonic
  - In(s) = $\bigcap_{\textbf{s' 5 pred(s)}}$ Out(s')
  - Temp = Gen(s) ^ (In(s) – Kill(s))
- Putting the two together
  - Temp = f_s ($\bigcap_{\textbf{s' 5 pred(s)}}$ Out(s'))

Termination

- We know algorithm terminates because
  - The lattice has finite height
  - The operations to compute In and Out are monotonic
  - On every iteration we remove a statement from the worklist and/or move down the lattice.